

## *Resource Depletion, Factor Proportions, and Trade*

Henry Thompson

February 2017

This paper develops a dynamic, factor proportions, small open economy with a resource intensive export. Optimal depletion implies the resource stock is treated as an asset and depleted so price rises at the rate of the capital return. Capital grows with saving, and labor at a constant rate. Depletion, export production, and the capital return fall as import competing production and the wage rise. The paper compares the effects of taxes on imports, exports, and depletion. It also examines the alternative assumptions of a constant depletion rate, tragedy of the commons, and myopic resource owner.

Keywords: nonrenewable resource, depletion, production, trade

Thanks to Andy Barnett and Farhad Rassekh for discussion on some critical points. Henry Kinnucan, Gilad Sorek, and Aditi Sengupta also provided useful comments.

Contact: Economics Department, Auburn University, AL 36849, [henry.thompson@auburn.edu](mailto:henry.thompson@auburn.edu), 334-844-2910, Skype [henry.thompson.auburn](https://www.skype.com/en/contacts/henry.thompson.auburn)

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Nonrenewable resources provide the foundation for exports and income in many countries. Consider a small open economy depleting a nonrenewable resource for a resource intensive export. High transport cost precludes direct export of the resource. The resource is combined with capital and labor to produce the export and an import competing good. This structure describes a small, resource abundant, developing country with nonrenewable hydrocarbon or mineral resources. The critical issue is how to maintain or increase income as the nonrenewable resource is depleted.

The present model integrates factor proportions production, resource economics, and economic growth. With optimal depletion, the resource is depleted so its price grows at the rate of the capital return. Capital grows with investment from saving, and labor at a constant rate. The general equilibrium of production involves the dynamics of depletion as well as the wage, capital return, the two outputs, and income.

Resource input diminishes with its rising price as the resource stock is depleted. The economy moves toward labor intensive production with the wage rising as capital return falling. In spite of the declining gains from trade, income can increase with sufficient investment. Taxes on depletion, outputs, and factor prices have different transitory effects with long term trends resuming.

The following section reviews the related literature and previews the model. The second section introduces the dynamics followed by a section with a short review of substitution in production. Sections 4 and 5 then present the model followed by a section on taxing trade and another on taxing depletion. Section 8 presents simulations with log linear production functions,

followed by a section examining the alternative assumptions of a constant depletion rate, tragedy of the commons, and myopic resource owner.

## **1. A review of the related literature**

The present model builds on factor proportions production with elements of resource economics and growth theory. The general equilibrium, factor proportions model assumes full employment of factors and competitive pricing of goods. The present model with three factors and two goods is developed by Ruffin (1981), Jones and Easton (1983), and Thompson (1985). The assumption of optimal depletion ties the resource price to the capital return, simplifying the structure of the model. The factor supply dynamics, however, complicate the analysis with a shifting production frontier.

Optimal depletion theory treats the stock of the nonrenewable resource as an asset equivalent to capital in the literature including Dixit, Hammond, and Hoel (1980), Hamilton (1995), Withagen and Asheim (1998), and Sato and Kim (2002). The resource owner depletes to keep its price rising at the rate of the capital return assuming a perfect asset market. The resource enters the utility function in the optimal depletion literature, while the present model treats the resource as a factor of production.

Economic growth with optimal depletion of a nonrenewable resource as in Stiglitz (1974), Dasgupta and Heal (1979), and Solow (1974, 1986) has a single output. Hartwick (1977) shows income is maintained in the model with resource and capital inputs if all resource income is invested. In the model including labor, Thompson (2012) finds that resource and income must be invested. The present paper expands this three factor growth model to two goods.

The “Dutch disease” literature including Corden and Neary (1982) focuses on a booming resource sector and declining manufacturing, capturing the situation of a developed country with

a resource discovery. In the present model, depletion of the known resource stock and investment move the economy away from exporting the resource intensive good in a situation more akin to a resource abundant developing country.

The growth model of van Geldrop and Withagen (1993) assumes capital and a single output traded at world prices. In the present model, a fixed exogenous capital return would imply a constant percentage increase in the resource price and faster depletion. The falling capital return in the present model implies a convex resource price path and slower depletion.

Bretscher and Valente (2012) and Gaitan and Roe (2012) analyze the dynamics of the terms of trade for a resource exporting country. For a given worldwide stock, the terms of trade for a resource exporter would improve but the level of trade would diminish. The present small open economy exports a resource intensive good at fixed terms of trade and is able to move toward other production through investment.

The two sector growth model of Uzawa (1963) and Takayama (1963) has produced capital goods in one sector. The other sector produces consumer goods with capital and labor inputs. The present model assumes capital goods are a costless transformation of income as in neoclassical growth theory.

## **2. The basics of the present dynamic factor proportions model**

The underlying structure of the present dynamic model is the general equilibrium of two traded goods produced with three factors of production. Labor  $L_t$  grows at the constant rate  $\lambda$ . The capital stock  $K_t$  grows with investment based on the saving rate  $\sigma$ . Income  $Y_t$  is determined as the sum of payments to the three factors or the sum of the value of the two outputs. Capital and labor are combined with the resource  $N_t$  to produce the two outputs  $x_{jt}$  traded at exogenous world prices  $p_j$  for  $j = 1, 2$ . Depletion  $N_t$  reduces the resource stock  $S_t$ .

The three factors are mobile between sectors and paid marginal products. The evolving wage  $w_t$  and capital return  $r_t$  endogenously clear those factor markets. The resource owner optimally depletes to ensure price  $n_t$  increases at the rate of the capital return  $r_t$ . While resource markets are often characterized by supply disruptions, underlying price trends tend to resurface.

Factor intensity is critical to the evolving economy, as is substitution between the three factors. Capital and labor are fully employed. The two goods are competitively produced with average cost equal to price. Neoclassical production functions exhibit constant returns, the production structure of factor proportions theory. The factor price and output paths depend on factor intensity, substitution, and the state of the economy.

### 3. Labor growth, investment, and optimal depletion

The growth of labor and capital is based on neoclassical growth theory. Labor  $L_t$  grows at the constant rate  $\lambda \equiv L_t'/L_t$  where the prime ' represents a time derivative,  $L_t' = dL_t/dt$ .

The instantaneous change in capital  $K_t' = dK_t/dt$  equals investment assuming no depreciation. The constant saving rate  $\sigma$  implies  $K_t' = \sigma Y_t$  where  $Y_t$  is income. Capital is transformed from capital without cost.

Capital  $K_t$  is fully utilized in the two sectors according to  $K_t = \sum_j K_{jt} = \sum_j a_{Kjt} x_{jt}$  where  $a_{Kjt}$  is the flexible cost minimizing input of capital per unit of output  $x_{jt}$  at time  $t$ . Labor is fully employed according to  $L_t = \sum_j L_{jt} = \sum_j a_{Ljt} x_{jt}$ . The resource enters production in both sectors according to  $N_t = \sum_j N_{jt} = \sum_j a_{Njt} x_{jt}$ .

Depletion  $N_t$  reduces the resource stock  $S_t$  according to  $N_t = -S_t'$ . Optimal depletion implies the Hotelling (1931) condition that equates the rate of return on the resource stock to the capital return,  $n_t'/n_t = r_t$  implying the asset market clearing condition,

$$n_t' = r_t n_t. \tag{1}$$

Transport costs are assumed too high for the resource rich country to export the resource directly. Where  $n_t^*$  is the global price of the resource and  $T$  its transport cost, the maintained assumption is  $n_t^* - T < n_t$ .

Income is the payment to domestic factors with constant returns and competitive factor markets according to the Euler theorem,  $Y_t = r_t K_t + w_t L_t + n_t N_t$ . Factors are paid marginal products in both sectors assuming free mobility. Income is equivalently output  $Y_t = \sum_j p_j x_{jt}$  at exogenous world prices  $p_j$  implying  $Y_t' = \sum_j p_j x_{jt}'$ .

#### 4. A summary of substitution in production

The cost minimizing input mix adjusts to changing factor prices as developed by Allen (1938) and Takayama (1982). Three inputs include the possibility of complements and elastic substitutes analyzed by Thompson (2006). Assume the neoclassical production functions  $x_{jt} = x_j(K_{jt}, L_{jt}, N_{jt})$  with constant returns to scale. Depletion changes according to  $N_t' = \sum_j x_{jt} a_{Njt}' + \sum_j a_{Njt} x_{jt}'$ . Homothetic production implies the flexible unit inputs  $a_{Nj}$  are functions of factor prices only.

Where  $S_{ikt}$  represents the evolving substitution of factor  $i$  relative to the price of factor  $k$  at time  $t$ , adjustment in the resource input expands to

$$N_t' = S_{Nrt} r_t' + S_{Nwt} w_t' + S_{Nnt} n_t' + \sum_j a_{Njt} x_{jt}', \quad (2)$$

with similar terms for capital substitution  $S_{Kit}$  and labor substitution  $S_{Lit}$ . For the resource, cross price substitution relative to the capital return is  $S_{Nrt} \equiv \sum_j x_{jt} (a_{Njt}' / r_t')$  and relative to the wage  $S_{Nwt} \equiv \sum_j x_{jt} (a_{Njt}' / w_t')$  with the negative own price term  $S_{Nnt} \equiv \sum_j x_{jt} (a_{Njt}' / n_t')$ .

Cost minimization and Shephard's lemma imply the flexible unit inputs  $a_{ijt}$  are first order partial derivatives of the cost function with respect to factor prices. Negative own price substitution follows from concave cost functions. Assuming production is homogeneous of

degree one, the cost minimizing  $a_{ijt}$  are homogeneous of degree zero. Young's theorem implies symmetric substitution terms,  $S_{ikt} = S_{kit}$ .

## 5. The dynamic general equilibrium and factor intensity

The first equation in the system (3) below is capital utilization including the condition  $K_t' = \sigma Y_t$ . The second equation includes  $L_t' = \lambda L_t$  in the labor employment condition. The resource utilization condition (2) in the third equation includes the optimal price change  $n_t'$  from (1).

The last two equations in (3) are competitive pricing of the goods. Price equals cost,  $p_j = a_{Kj}r_t + a_{Lj}w_t + a_{Nj}n_t$  for good  $j$  where  $p_j$  is given for the small open economy. Differentiate and simplify with the cost minimizing envelope condition to find  $p_j' = a_{Kjt}r_t' + a_{Ljt}w_t' + a_{Njt}n_t'$ .

In the dynamic system of instantaneous changes,

$$\begin{pmatrix} S_{Krt} & S_{Kwt} & 0 & a_{K1t} & a_{K2t} \\ S_{Lrt} & S_{Lwt} & 0 & a_{L1t} & a_{L2t} \\ S_{Nrt} & S_{Nwt} & -1 & a_{N1t} & a_{N2t} \\ a_{K1t} & a_{L1t} & 0 & 0 & 0 \\ a_{K2t} & a_{L2t} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r_t' \\ w_t' \\ N_t' \\ x_{1t}' \\ x_{2t}' \end{pmatrix} = \begin{pmatrix} \sigma Y_t - S_{Knt}r_t n_t \\ \lambda L_t - S_{Lnt}r_t n_t \\ -S_{Nnt}r_t n_t \\ p_1' - a_{N1t}r_t n_t \\ p_2' - a_{N2t}r_t n_t \end{pmatrix}, \quad (3)$$

changes in world prices  $p_1'$  and  $p_2'$  are set to zero for the small open economy. Endogenous instantaneous changes  $r_t'$ ,  $w_t'$ ,  $N_t'$ , and  $x_{jt}'$  depend on factor intensity, substitution, and the state of the economy in the levels of  $r_t$ ,  $n_t$ ,  $Y_t$ , and  $L_t$ . The change in income  $Y_t'$  is derived separately. The model is solved by Cramer's rule with its negative determinant  $\Delta_t = -(a_{K1t}a_{L2t} - a_{L1t}a_{K2t})^2 < 0$ .

The resource is most intensive or extreme in producing exported good 1 with labor extreme for import competing good 2. Capital is the middle factor in the intensity condition,

$$a_{N1t}/a_{N2t} > a_{K1t}/a_{K2t} > a_{L1t}/a_{L2t}. \quad (4)$$

Three terms summarize this factor intensity,

$$a_{NKt} \equiv a_{N1t}a_{K2t} - a_{K1t}a_{N2t} > 0$$

$$a_{NLt} \equiv a_{N1t}a_{L2t} - a_{L1t}a_{N2t} > 0 \quad (5)$$

$$a_{KLt} \equiv a_{K1t}a_{L2t} - a_{L1t}a_{K2t} > 0 .$$

The resource is intensive in good 1 relative to both capital and more so relative to labor. Labor is intensive in good 2 relative to capital, and more so relative to the resource. The middle factor capital is intensive in good 1 relative to labor, but intensive in good 2 relative to the resource.

## 6. Endogenous factor prices, depletion, and production

The directions of evolving changes in the capital return and wage solving (3) depend only on factor intensity,

$$r_t' = -a_{NLt}r_t n_t / a_{KLt} < 0 \quad (6)$$

$$w_t' = a_{NKt}r_t n_t / a_{KLt} > 0 .$$

The wage rises given factor intensity (5) as the declining depletion favors import competing production and its extreme labor input. The return  $r_t$  to the middle factor capital diminishes as its supply increases.

If labor were the middle factor, the signs in (6) would be reversed. Capital would then benefit as the intensive factor in the expanding import competing industry. If the resource were the middle factor, both changes in (6) would be negative. The declining resource input would then lower productivities of both other factors.

The directions of change in (6) are independent of the evolving levels of capital and labor, the factor price equalization property between these two factors. The capital/labor ratio  $K_t/L_t$  in the economy depends on the rates of labor growth and saving as well as the state of the economy, rising if  $Y_t/K_t > \lambda/\sigma$ .

The second order effect of the resource price  $n_t''$  depends on factor intensity as well as the levels of  $n_t$  and  $r_t$  in the condition  $n_t'' = r_t n_t' + n_t r_t' = (a_{KLt}r_t - a_{NLt}n_t)r_t n_t / a_{KLt}$ . From (5)  $a_{NLt} > a_{KLt}$

implying the path of the resource price is increasing concave if  $n_t \geq r_t$  as would occur eventually. At lower levels of  $n_t$  in the early stages of depletion, however, the resource price path could be increasing convex.

Solving the system (3) for the change in depletion,

$$N_t' = -r_t n_t \Delta_{32} / \Delta_t < 0, \quad (7)$$

where  $\Delta_{32}$  is the determinant of the factor proportions model with three factors and two goods. Neoclassical production and cost minimization imply  $\Delta_{32} < 0$  as developed by Chang (1979) and Thompson (1985). The familiar assumption of declining depletion in partial equilibrium is due to the neoclassical production functions. Resource demand is downward sloping regardless output effects, factor prices, factor intensity, or substitution.

Output of exported good 1 evolves according to

$$x_{1t}' = [a_{KLt} \kappa_t - r_t n_t (a_{NKt} S_{1t} + a_{NLt} S_{2t} + a_{KLt} S_{3t})] / a_{KLt}^2, \quad (8)$$

where  $S_{1t} \equiv a_{L2t} S_{Kwt} - a_{K2t} S_{Lwt} > 0$ ,  $S_{2t} \equiv a_{K2t} S_{Klt} - a_{L2t} S_{Krt} > 0$ ,  $S_{3t} \equiv a_{L2t} S_{Knt} - a_{K2t} S_{Lnt}$ , and  $\kappa_t \equiv a_{L2t} \sigma Y_t - a_{K2t} \lambda L_t$ . There is a presumption that  $x_{1t}' < 0$  but the direction of change depends on factor intensity, substitution, and the state of the economy. High saving and low labor growth favor an increase in  $x_{1t}$ . The opposite presumption for import competing output is  $x_{2t}' > 0$ .

The change in income is  $Y_t' = x_{1t}' + x_{2t}'$  given  $p_1' = p_2' = 0$ . As suggested by (8) the change in income  $Y_t'$  depends on factor intensity, substitution, and the state of the economy. Higher saving favors increases in both outputs. While the rising wage and resource price favor rising income, the declining capital return favors a decrease. A larger supply of labor favors rising income. A higher level of depletion favors rising income as well. A higher capital stock, however, favors falling income with the falling capital return. In a developing country with a low level of capital

and high level of labor, income would more likely rise. Income per worker rises under the condition  $Y_t' > \lambda Y_t$ .

## 7. The production frontier and trade

Adjustments in production and trade are illustrated in Figure 1. Homothetic utility implies the constant consumption ratio  $c_1/c_2$  at exogenous world prices. Production occurs at point  $P_0$  on the production frontier with utility maximizing consumption at  $C_0$ . The exogenous terms of trade  $tt$  for the small open economy imply balanced trade on the trade triangle. The height of the terms of trade line  $tt$  connecting  $P_0$  and  $C_0$  is a gauge of income.

\* Figure 1 \*

The production frontier shifts due to changing factor supplies with depletion, investment, and labor growth. For discussion, assume the production point moves northwest in Figure 1 with falling export production and rising import competing production. Income increases with the production point moving above the  $tt$  line in Figure 1 if the increase in the value of import competing production outweighs the decrease in the value of export production. That is, income rises if  $x_{2t}' > -x_{1t}'$ . If the new production point were below the  $tt$  line, income would fall.

## 8. Taxes on trade and depletion

Tariffs or taxes on trade change domestic prices of the two goods. Exogenous world prices are unaffected. For simplicity assume  $p_1 = p_2 = 1$ . The price of the import with tariff rate  $t$  is then  $(1 + t)$ . A change in the tariff changes domestic price by  $dt$ . Solving the system (3) for this effective change in the price of imported good 2,

$$\begin{aligned} r_t'/dt &= r_t' - a_{L1t}/a_{KLt} < 0 \\ w_t'/dt &= w_t' + a_{K1t}/a_{KLt} > 0, \end{aligned} \tag{9}$$

where  $r_t' < 0$  and  $w_t' > 0$  are the evolving factor price trends in (6).

The impacts of a change in the tariff are the products of the partial derivatives in (9) and  $t'$ . An increase in the tariff rate reinforces the rising wage and falling capital return. If labor were intensive in import production relative to capital, the tariff would work in opposite directions with the underlying trends resuming.

The effect of the tariff on depletion depends on substitution,

$$N_t'/dt = N_t' - (a_{NKt}S_{4t} + a_{NLt}S_{5t} + a_{KLt}S_{6t})/a_{KLt}^2, \quad (10)$$

where  $S_{4t} \equiv a_{L1t}S_{Kwt} - a_{K1t}S_{Lwt} > 0$ ,  $S_{5t} \equiv a_{K1t}S_{Krt} - a_{L1t}S_{Krt} > 0$ , and  $S_{6t} \equiv a_{L1t}S_{Nrt} - a_{K1t}S_{Nwt}$ . The negative term  $N_t' < 0$  in (10) is the underlying diminished depletion from (7). The tariff is expected to lower depletion by reducing the level of trade. An increase in depletion is favored, however, by a negative  $S_{6t}$  with the resource a strong substitute relative to the rising wage and a weak substitute or complement relative to the rising capital return.

An export tax or subsidy changes its price inside the economy. An export tax  $\tau$  reduces the price received by firms to  $(1 - \tau)p_1 = 1 - \tau$  while a subsidy raises the price to  $1 + \tau$  assuming  $p_1 = 1$ . Solving the system (3) for a change  $d\tau$  in the export subsidy or tax, the capital return adjusts according to

$$r_t'/d\tau = r_t' + a_{L2t}/a_{KLt}, \quad (11)$$

where  $r_t' < 0$  is the underlying negative trend in (6). Combining (6) and (11) the change the export price has a transitory positive effect on the capital return assuming  $r_t n_t < a_{L2t}/a_{LNt}$ . An export subsidy would have a positive effect on the capital return given low levels of  $r_t$  and  $n_t$  and low resource intensity relative to labor in export production. Regardless of any transitory effect, the negative trend in  $r_t$  resumes.

The wage is affected by the export subsidy/tax in the system (3) according to

$$w_t'/d\tau = w_t' - a_{K2t}/a_{KLt}, \quad (12)$$

where  $w_t' > 0$  is the underlying positive trend from (6). From (6) and (12) an increase in the export price has a negative transitory effect on the wage if  $r_t n_t < a_{K2t}/a_{NKt}$ . An export subsidy then has a negative transitory effect on the wage given low levels of  $r_t$  and  $n_t$  and low intensity of the resource relative to capital in export production. Regardless, the underlying positive wage trend in (6) resumes. The effect of an export tax or subsidy on depletion  $N_t$  depends on substitution in an expression similar to (10).

The effects of tariffs and subsidies on outputs are captured by their price effects. For instance, the effect of a change in the export price on its output,

$$x_{1t}'/p_1' = (2a_{K2t}a_{L2t}S_{Kwt} - a_{K2t}^2S_{Lnt} - a_{L2t}^2S_{Krt}) + x_{1t}', \quad (13)$$

includes the presumed underlying negative trend  $x_{1t}'$  in (8). Expressions for  $x_{2t}'/p_1'$ ,  $x_{1t}'/p_2'$ , and  $x_{2t}'/p_2'$  are similar. There is a presumption outputs are concave in price changes but factor supplies shift clouding the result. Regardless of the transitory effects, the underlying trends would resume.

A depletion tax raises the price  $n_t$  of the resource to  $(1 + t_N)n_t$  as an input in production, lowering its cost minimizing unit inputs. Resource demand falls lowering the level of depletion  $N_t$ . The resource owner receives a smaller payment and sells less of the resource. At the higher price, there is a larger evolving decrease in depletion  $N_t'$  from (7). Changes in the capital return and wage in (6) are amplified with labor benefiting from the shift toward import competing production. Stronger substitution  $S_{Lr}$  of labor relative to the falling price of capital favors this shift in production, as do a lower saving rate  $\sigma$  and higher labor growth rate  $\lambda$ .

## 9. Simulated time paths

The following simulations illustrate time paths over ten time periods with the log linear Cobb-Douglas production functions,

$$x_{1t} = K_{1t}^{0.6} L_{1t}^{0.1} N_{1t}^{0.3} \quad (14)$$

$$x_{2t} = K_{2t}^{0.4} L_{2t}^{0.5} N_{2t}^{0.1} .$$

The factor intensity condition (4) holds across the present simulations implying the rising wage and falling capital return in (6).

Exogenous world prices of the two goods are  $p_1 = p_2 = 0$ . The saving rate is  $\sigma = 0.25$  and labor growth rate  $\lambda = 0.01$ . Initial values of capital and labor are  $K_1 = 100,000$  and  $L_1 = 100$ . Assuming the resource price  $n_1 = 24$  implies the level of depletion  $N_1 = 9.5$  in the initial equilibrium. Variables are rescaled for presentation in the Figures.

The resource price rises according to  $n_{t+1} = (1 + r_t)n_t$  determining depletion  $N_{t+1}$ . The baseline economy in Figure 2 trends toward production of import competing good 2 as depletion  $N_t$  decreases at a decreasing rate. Income  $Y_t$  falls but only slightly as the rising wage  $w_t$  and resource price  $n_t$  nearly offset the declining capital return  $r_t$ . The level of trade falls as the economy faces a bleak future depleting its resource.

\* Figure 2 \*

Figure 3 presents the more frugal scenario of a higher saving rate  $\sigma = 0.40$  and lower labor growth rate  $\lambda = 0.005$ . Export production  $x_{1t}$  increases as investment more than offsets the slower decline in depletion  $N_t$ . Import competing production  $x_{2t}$  expands but more slowly than in Figure 2. Trends for the wage  $w_t$  and capital return  $r_t$  are identical to those in Figure 2 due to the factor price equalization property for those two factors. Income  $Y_t$  increases as does consumption of each good and perhaps the level of trade.

\* Figure 3 \*

Figure 4 illustrates the effects of a depletion tax  $t_N = 10\%$  in period 4 on the baseline economy of Figure 2. Depletion  $N_t$  falls as the resource price rises to  $1.1n_t$  along the higher

depletion trend. Resource intensive output  $x_{1t}$  and the capital return  $r_t$  both fall before resuming negative trends. The wage  $w_t$  and labor intensive output  $x_{2t}$  increase as labor and capital are released from export production. There is a negative effect on income due to any tax or subsidy with the decreasing trend in the baseline economy of Figure 2 resumes.

\* Figure 4 \*

Figure 5 shows the effects of an import tariff of  $t = 10\%$  on the baseline economy in Figure 2. The tariff reinforces underlying trends in the wage  $w_t$  and capital return  $r_t$ . Depletion  $N_t$  and export production  $x_{1t}$  both fall to lower trends as import competing output  $x_{2t}$  jumps to a higher trend. An export tax lowers the price received by firms in the industry and has similar effects on the economy.

\* Figure 5 \*

Figure 6 shows the effects of an export subsidy of 5%. Export production  $x_{1t}$  jumps with depletion  $N_t$  as both overshoot new trends. Production of import competing  $x_{2t}$  and the wage  $w_t$  similarly overshoot. The effects of the export subsidy weaken the following period at  $t = 5$  and reverse at  $t = 6$  before resuming new trends at  $t = 7$  as the rising resource price  $n_t$  overcomes the higher export price. The capital return  $r_t$  rises temporarily before resuming its decline, accounting for the negligible change in the resource price  $n_t$ .

\* Figure 6 \*

## 10. Alternative assumptions on depletion

The assumption of a constant depletion rate is justified due to the simplicity it affords partial equilibrium analysis. A fraction  $\alpha$  of the resource stock  $S_t$  is depleted each period according to  $N_t = \alpha S_t$ . The condition  $(N_t/S_t)' = 0$  implies  $N_t' = -\alpha N_t$  in a condition added to the system (3) to allow the endogenous  $n_t'$ . A higher depletion rate  $\alpha$  would imply higher  $N_t'$ ,  $n_t'$ , and

$r_t'$ , but lower  $w_t'$ . A higher saving rate  $\sigma$  raises capital growth and  $w_t'$  but lowers  $r_t'$  and  $n_t'$  as capital replaces the resource in export production. Higher labor growth  $\lambda$  would result in lower  $w_t'$  but higher  $r_t'$  and  $n_t'$ . Other evolving changes depend on substitution. Taxes on imports and exports have the unambiguous expected output effects. The other effects, however, are not simplified relative to the optimal depletion model.

A tragedy of the commons implies the resource is priced at marginal extraction cost  $E_t$ . Constant  $E_t$  implies  $n_t' = 0$  eliminating  $r_t n_t$  from the exogenous vector in (3). The constant resource price implies  $w_t$  and  $r_t$  are also constant. The evolving export production in (8) simplifies to  $x_{1t}' = (a_{L2t}\sigma Y_t - a_{K2t}\lambda L_t)a_{K1t}^{-1}$ . A higher saving rate and lower labor growth rate favor export production  $x_{1t}$  and reduced import competing production in  $x_{2t}' = (a_{K1t}\lambda L_t - a_{L2t}\sigma Y_t)a_{K1t}^{-1}$ . Depletion increases according to  $N_r' = \sum_j a_{Nrt} x_{jt}' = (a_{N1t}\sigma Y_t + a_{N2t}\lambda L_t)a_{K1t}^{-1} > 0$  reflecting the tragedy. Income rises due to the gains from competition according to  $Y_t' = w_t L_t' + r_t K_t' + n_t N_t' = [(w_t a_{K1t} + a_{N1t})\lambda L_t + (r_t a_{K1t} + a_{N1t})\sigma Y_t]a_{K1t}^{-1} > 0$ . Increasing marginal extraction cost as a function of the resource stock leads properties similar.

A myopic resource owner maximizes immediate profit disregarding the asset value of the resource stock setting marginal revenue  $R_t$  equal to marginal extraction cost  $E_t$ . Total resource revenue  $n_t N_t$  implies  $R_t = (n_t N_t)' / N_t' = n_t + N_t n_t' / N_t' = E_t$  and  $n_t' = N_t'(E_t - n_t) / N_t$ . The resource price  $n_t$  has to be greater than  $E_t$  implying  $n_t$  and  $N_t$  move in opposite directions. The myopic resource owner suffers a falling share of income.

## 11. Conclusion

In the present setting, depletion of the nonrenewable resource diminishes as the small open economy trends away from resource intensive export. In spite of the declining gains from trade, income can increase due to investment given the flexibility in outputs and factor prices.

Taxes on import, export, and depletion bump factor prices but underlying trends resume. An export subsidy causes opposite factor price shifts overshooting the resumed trends.

Possible extensions and applications of the present model include an endogenous rate of saving depending on the level of income. Saving behavior could also depend on intertemporal utility maximization for the two goods. The growth rate of labor could be allowed to decrease in income. Capital goods could be separated in production, or as a separate import. Depletion and trade with an exogenous international capital return could be analyzed. The resource could be renewable, leading to conditions that would sustain a perpetual stock. Two large economies would lead to improved terms of trade for the resource intensive exporter. Simulations with flexible production functions allowing variation in factor shares of income would lead to varied time paths. The minimal saving rate to maintain income can be estimated, and the long term effects of taxes and subsidies simulated.

## References

- Allen, R.G.D. (1938) *Mathematical Analysis for Economists*, New York: St. Martin's Press
- Bretschger, Lucas and Simone Valente (2012) Endogenous growth, asymmetric trade and resource dependence, *Journal of Environmental Economics and Management* 64, 301-11.
- Chang, Winston (1979) Some theorems of trade and general equilibrium with many goods and factors, *Econometrica* 47, 790-26.
- Corden, Max and Peter Neary (1982) Booming sector and de-industrialisation in a small open economy, *The Economic Journal* 92, 825-48.
- Dasgupta, P.S. and G.M. Heal (1974) The optimal depletion of exhaustible resources, *Review of Economic Studies* 41, 3–28.
- Dixit, A., P. Hammond, and M. Hoel (1980) On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion, *Review of Economic Studies* 47, 551-6.
- Gaitan, Beatriz and Terry Roe (2012) International trade, exhaustible-resource abundance and economic growth, *Review of Economic Dynamics* 15, 72-93.
- Hamilton, Kirk (1995) Sustainable development, the Hartwick rule, and optimal growth, *Environmental and Resource Economics* 5, 393-411.
- Hartwick, J. (1977) Intergenerational equity and the investing of rents from exhaustible resources, *American Economic Review* 66, 972-74
- Hotelling, H. (1931) The economics of exhaustible resources, *Journal of Political Economy* 39, 137-75.
- Jones, Ron and Stephen Easton (1983) Factor intensities and factor substitution in general equilibrium, *Journal of International Economics* 15, 65-99.
- Robinson, T. J. C. (1980) Classical foundations of the contemporary economic theory of nonrenewable energy resources, *Resources Policy* 6, 278-89.
- Ruffin, Roy (1981) Trade and factor movements with three factors and two goods, *Economics Letters* 7, 177-82.
- Sato, Ryuzo and Youngduk Kim (2002) Hartwick's rule and economic conservation laws, *Journal of Economic Dynamics and Control* 26, 437-49.
- Solow, Robert (1974) Intergenerational equity and exhaustible resources, *Review of Economic Studies* 41, 29-45.

Solow, Robert (1986) On the intergenerational allocation of natural resources, *Scandinavian Journal of Economics* 88, 141-49.

Stiglitz, Joseph (1974) Growth with exhaustible natural resources: Efficient and optimal growth paths, *Review of Economic Studies* 41, 123-37.

Takayama, Akira (1963) On a two sector model of economic growth – A comparative static analysis, *Review of Economic Studies* 30, 94-104.

Thompson, Henry (1985) Complementarity in a simple general equilibrium production model, *Canadian Journal of Economics* 18, 616-21.

Thompson, Henry (2006) The applied theory of energy substitution in production, *Energy Economics* 28, 410-25.

Thompson, Henry (2012) Economic growth with a nonrenewable resource, *Journal of Energy and Development* 36, 35-43.

Uzawa, Hirofumi (1963) On a two-sector model of economic growth, II, *Review of Economic Studies*, 30, 105-18.

van Geldrop, Jan and Cees Withagen (1993) General equilibrium and international trade with exhaustible resources, *Journal of International Economics* 34, 341-57.

Withagen, Cees and Geir Asheim (1998) Characterizing sustainability: The converse of Hartwick's rule, *Journal of Economic Dynamics and Control* 23, 159-63.

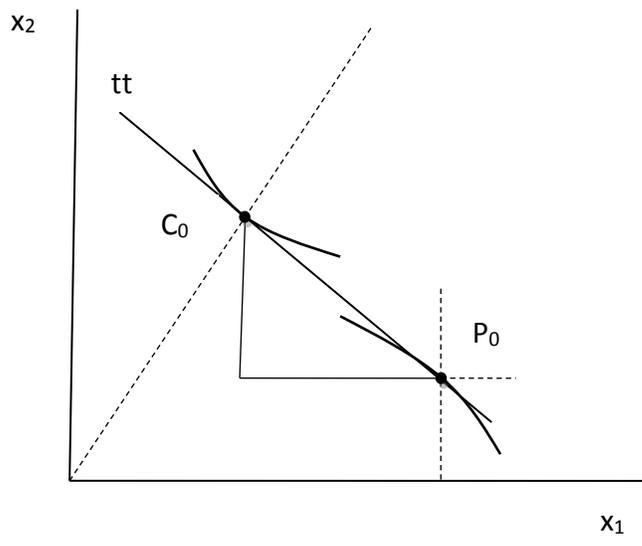


Figure 1. Production and Trade

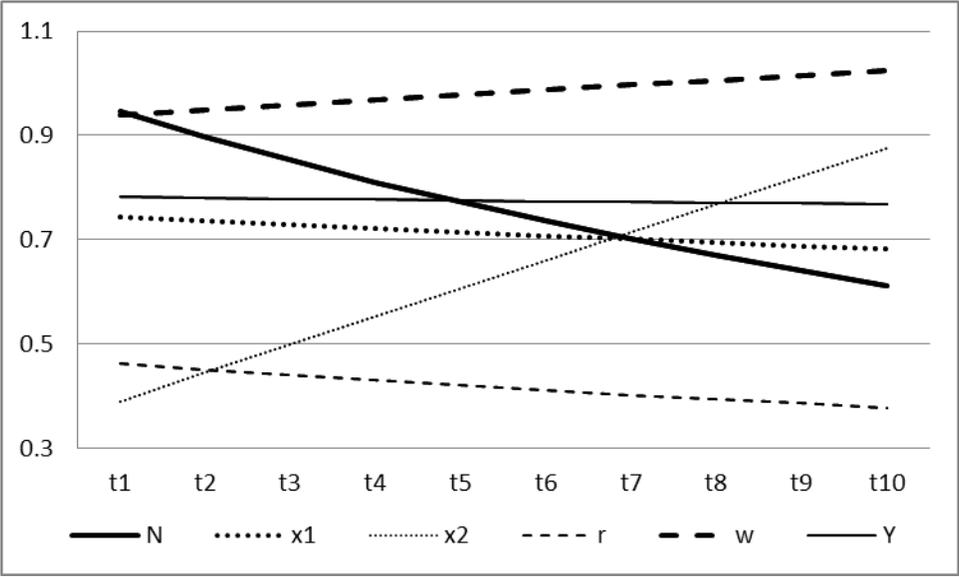


Figure 2. Baseline Cobb-Douglas production

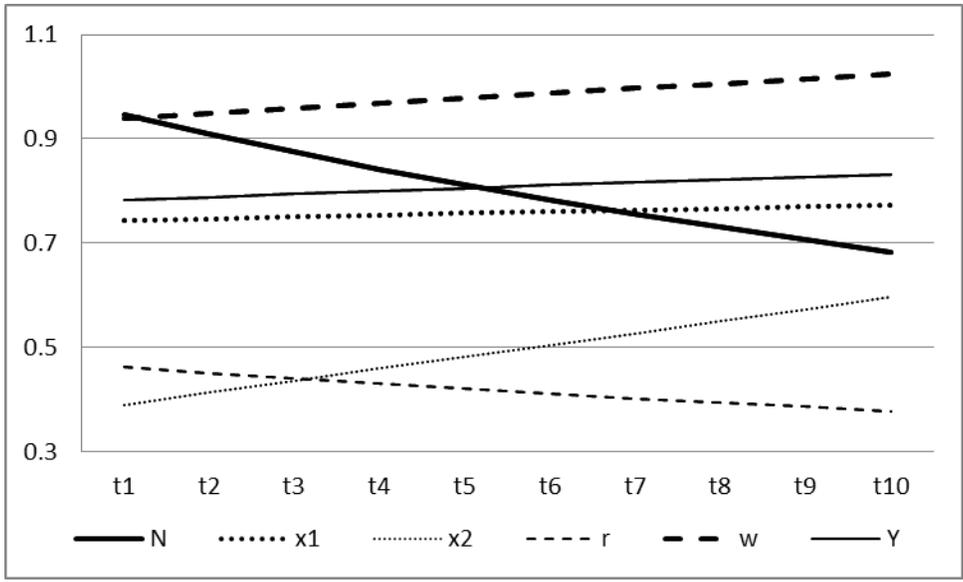


Figure 3. Rising income with high saving and low labor growth

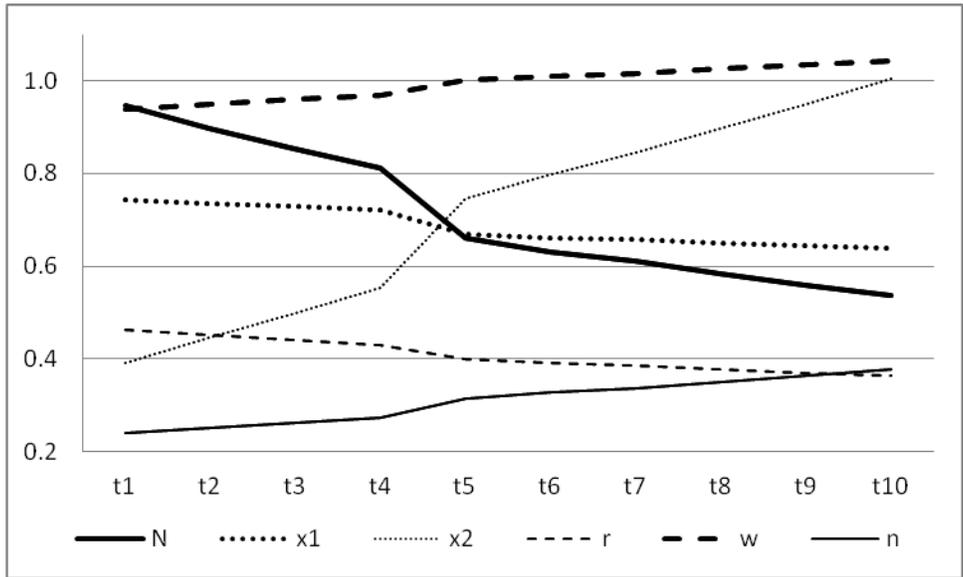


Figure 4. 10% depletion tax in the baseline economy

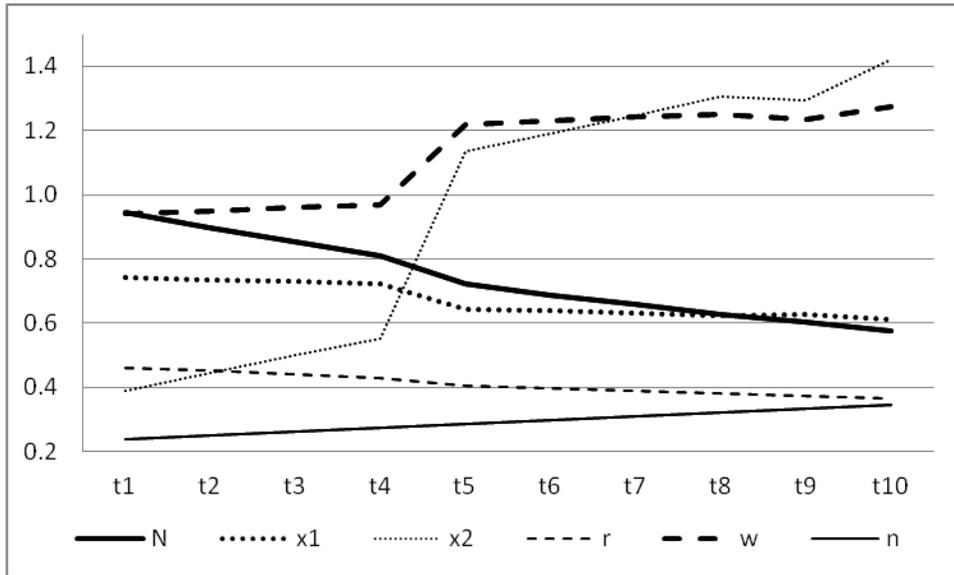


Figure 5. 10% import tariff

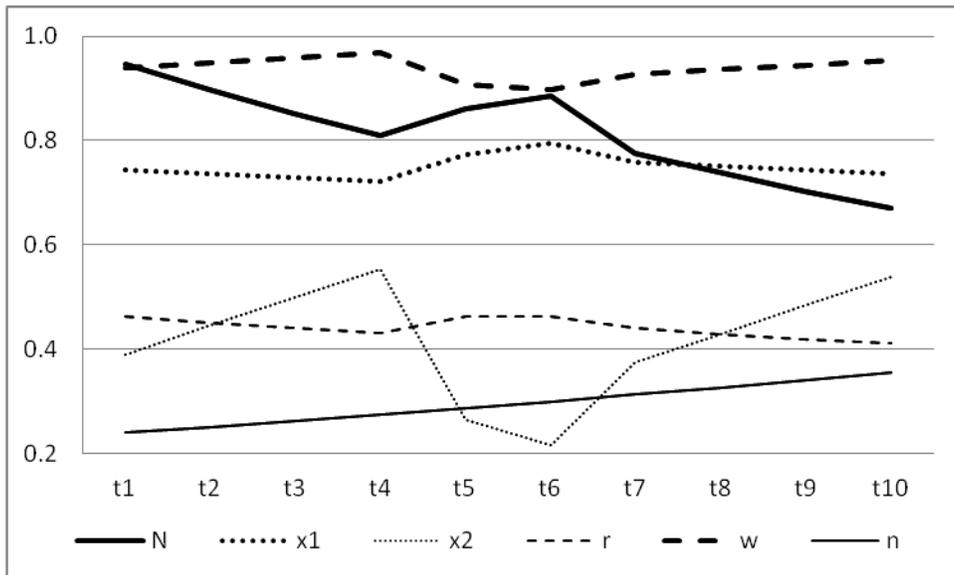


Figure 7. Overshooting with a 5% export subsidy